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2. Proposed by E. S. LOOMIS, A. M., Ph. D., Professor of Mathematics, Baldwin University, Berea, Ohio.

2. Show that the indeterminate form $\frac{x - \frac{2}{3} \sin x - \frac{1}{3} \tan x}{x^5} = \frac{-1}{20}$, when $x=0$.

[Ex. 51, p. 112, *Williamson's Differential Calculus*.]

I. Solution by C. W. M. BLACK, Department of Mathematics, Wilmington Conference Academy, Dover, Delaware, and G. SERPELL, Student at Virginia Military Institute, Lexington, Virginia.

$$\begin{aligned}\frac{f'(x)}{\phi'(x)} &= \frac{1 - \frac{2}{3} \cos x - \frac{1}{3} \sec^2 x}{5x^4} \\ \frac{f''(x)}{\phi''(x)} &= \frac{\frac{2}{3} \sin x - \frac{2}{3} \sec^2 x \tan x}{20x^3} \\ \frac{f'''(x)}{\phi'''(x)} &= \frac{\frac{2}{3} \cos x - \frac{2}{3} \sec^2 x \tan^2 x - \frac{2}{3} \sec^4 x}{60x^2} \\ \frac{f^{IV}(x)}{\phi^{IV}(x)} &= \frac{-\frac{2}{3} \sin x - \frac{2}{3} \sec^2 x \tan^3 x - \frac{1}{3} \sec^4 x \tan x}{120x} \\ \frac{f^V(x)}{\phi^V(x)} &= \frac{-\frac{2}{3} \cos x - \frac{1}{3} \sec^2 x \tan^4 x - \frac{8}{3} \sec^4 x \tan^2 x - \frac{1}{3} \sec^6 x}{120} \\ &= \frac{-6}{120} = \frac{-1}{20}, \text{ when } x=0.\end{aligned}$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

$$\begin{aligned}\text{We have } (x - \frac{2}{3} \sin x - \frac{1}{3} \tan x) \div x^5 &= x - \frac{2}{3} \left(x - \frac{x^3}{6} + \frac{x^5}{120} - \text{etc.} \right) \\ - \frac{1}{3} \left(x + \frac{x^3}{3} + \frac{x^5}{15} + \text{etc.} \right) \div x^5 &= \frac{-1}{20}, \text{ when } x=0.\end{aligned}$$

Also solved by Alfred Hume, P. H. Philbrick, J. F. W. Scheffer, H. C. Whitaker, G. B. M. Zerr, and P. S. Berg.

3. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

The product of two sides of a triangle is $6,000(k^2)$, the length of the bisector of the included angle is $60(b)$. What is the maximum area of the triangle, what is the greatest length of the third side, and what is the total area swept over by the triangle, the bisector remaining fixed? [Selected from *Philadelphia Call*, 26 May, 1890.]

Solution by the Proposer.

Denote one of the including sides by r , the bisected angle by 2θ , the angle the bisector makes with the base by ϕ . The area $= \frac{1}{2} k^2 \sin 2\theta$ and is therefore a maximum with $\sin 2\theta$. The segment of the base adjacent to r is $\frac{r}{k} \sqrt{k^2 - b^2}$

and therefore $k \sin \theta = \sqrt{k^2 - b^2} \sin \phi$. From this equation $\sin 2\theta = \frac{2b}{k^2} \sqrt{k^2 - b^2}$ when $\cos \phi = 0$. In this case the triangle is isosceles, the third side $= 2\sqrt{k^2 - b^2}$ and the area $= b\sqrt{k^2 - b^2}$.

By trigonometry, $(k^2 - b^2)r^2 - 2rbk^2 \cos \theta + k^2b^2 = 0$, which is the polar equation of the locus of one end of the base, the bisector being the polar axis and the vertex of the triangle the pole. It is a circle with a center on the